J. B. PROLLA, *Weierstrass-Stone, the Theorem*, Approximation & Optimization, Vol. **5**, Verlag Peter Lang, 1993, iv + 130 pp.

This short book presents a thorough study of variations, extensions, and generalizations of the classical Stone–Weierstrass theorem. It includes coverage from both a functional analytic and an algebraic point of view. However, for example, Müntz's classical theorem, which characterizes when the linear span of a sequence of monomials is dense in $C[0, 1]$, is beyond the scope of this discussion. The material is well-organized. The short chapters make the book easily readable and available not only to experts.

The book contains 13 short chapters. Chapter 1 presents the classical Stone–Weierstrass theorem for subalgebras A of the continuous, real-valued functions on X. In Chapter 2, this is generalized to vector-valued functions. Chapter 3 discusses R. T. Jewett's proof of von Neumann's variant of the Stone–Weierstrass theorem. In Chapter 4, an extension of the Stone–Weierstrass theorem for subsets is given. The concept of multipliers associated with a nonempty subset W of $C(X; E)$ plays a crucial role here. Chapter 5 is titled as "A Generalized Bernstein Theorem" and examines when a convex cone contained in $C^+(X)$ is dense in it. Chapter 6 deals with Ransford's argument for the proof of the Stone–Weierstrass theorem. By using Zorn's lemma, this avoids the use of a difficult result of Jewett that the uniform closure of a set A with a property called property V is a lattice. Chapter 7 discusses the Choquet–Deny theorem on the description of the uniform closure of a convex inf-lattice or a convex conic inf-lattice of functions in $C(X; \mathbb{R})$. This is applied to describe the uniform closure of the inf-lattice generated by a linear subspace of $C(X; \mathbb{R})$) which contains the constant functions. Some of these results have applications in economics. The Kakutani–Stone theorem is also proved on the description of the uniform closure of a lattice of functions in $C(X; \mathbb{R})$. This theorem is used in Chapter 8 to establish Jewett's generalization of von Neumann's theorem on the uniform closure of subsets having property V. Chapter 9 offers a proof of the Bishop–Machado theorem. Chapter 10 deals with the metric approximation property. All the results about approximation in $C(X; E)$ with the supremum norm can be generalized to the case where E is locally convex, whose topology is determined by a family Γ of seminorms p. However, when E is not locally convex, many Stone–Weierstrass type questions are still unanswered. Some results that could be generalized, at least when the space X is of finite covering dimension, are presented in Chapter 11. In Chapter 12, approximation processes in $C(X; E)$ are studied. Shisha–Mond- and Bohman–Korovkin-type theorems are also discussed. Chapter 13 treats the case of uniform approximation of bounded, continuous, real-valued functions defined on a space X which is not assumed to be compact.

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R. A. DEVORE AND G. G. LORENTZ, *Constructive Approximation*, Grundlehren der mathematischen Wissenschaften, Vol. **303**, Springer-Verlag, 1993, x + 449 pp.

Approximation theory deals with objects to approximate and approximations. This makes room for three levels: at level 1, one looks for the approximations. Books and papers on level 1 contain tables, graphs, algorithms for special functions, etc. At level 2, one studies relations between the approximated objects and their approximations: when do we have existence and unicity, rates of decrease, shapes of error functions, etc. At level 3, one investigates theoretical properties without explicit reference to any approximation process, but with roots in approximation, such as moduli of continuity and related topics, Haar systems and spaces, interpolation spaces, etc.